SECTION A (56 marks)

Answer ALL questions in this section. ALL working for each question must be shown clearly.

1. (a) Evaluate $\int_0^{\pi/2} \sin\left(\frac{\pi}{2} - x\right) dx$ by substitution method. (4 marks)

   (b) Find the equation of a curve which passes through the point (1,0) whose gradient function is $3x^2 - \frac{1}{x^2}$

   (3 marks)

2. (a) Find the inverse of the matrix

   $$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$ (3 marks)

   (b) Solve the following system of equations by using the inverse of matrix A obtained in (a) above

   $2y + x = 10$

   $2x - y = 5$

3. (a) Write $\frac{3+i}{4-3i}$ in standard form

   (2 marks)

   (b) Give that $z = i(3 + 4i)$, find $|z|$. (3 marks)

   (c) Express $1 + i$ in polar form (2 marks)

4. (a) If $n(A) = 60$, $n(B) = 50$, $n(C) = 18$, $n(A \cap B) = 30$, $n(B \cap C) = 10$, $n(A \cap C) = 6$, $n(A \cap B \cap C) = 4$ Find $n(A \cup B \cup C)$ (3 marks)

   (b) In a class of 34 girls, 21 play tennis and 18 play netball. If all the girls play at least one of these games, how many play both netball and tennis?

   (4 marks)

5. (a) If $y = \frac{1 + \sin\theta}{\cos\theta}$, show that $\frac{1}{y} = \frac{1 - \sin\theta}{\cos\theta}$

   (3 marks)

   (b) If $s = \sin\theta$ and $c = \cos\theta$; simplify:

   (i) $\frac{s}{\sqrt{(1 - s^2)}}$

   (ii) $\left[\sqrt{1 - c^2}\right] s + c\sqrt{(1 - s^2)}$ (4 marks)

6. (a) Given that

   $$f(x) = \begin{cases} -2 & \text{for } x > 1 \\ 1 & \text{for } x = 1 \\ 3 & \text{for } x < 1 \end{cases}$$

$\frac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\cos 2x = 2\cos^2 x - 1$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

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(i) Sketch the graph of this function .

(ii) Evaluate f(- 5) (4 marks)

(b) Solve for t, $\left| 3 - 2t \right| < 5$ (3 marks)

7. (a) Find the values of a and b if the expression $x^4 + ax^3 + bx^2 - 4$ is exactly divisible by $x^2 - 9$. (4 marks)

(b) Determine the factors of the expression $x^2 - 2x - 3$. (3 marks)

8. (a) The probability that Halima will pass Mathematics is 0.4 and that Perpetua will do the same is 0.7. Find the probability that:
(i) both will pass Mathematics
(ii) Halima or Perpetua will pass Mathematics. (4 marks)

(b) Find x in $\begin{pmatrix} x \\ 2 \end{pmatrix} = 3$ when $\begin{pmatrix} n \\ r \end{pmatrix} = \dfrac{n!}{(n-r)!r!}$ (3 marks)

SECTION B (44 marks)

Answer ANY FOUR (4) questions from this section. ALL workings for each question answered must be shown clearly.

9. (a) The 4th, 6th and 9th terms of an arithmetic progression forms the first three terms of a geometric progression. Determine the common ratio of the geometric progression. (7 marks)

(b) Determine the equation of the tangent to the circle $x^2 + y^2 - 4x + 6y - 77 = 0$ when the tangent passes through the point of tangency at (5,6). (4 marks

10 (a) Show that $z^3 + 2z - 3 = (z - 1)(z^2 + 2 + 3)$ and hence find the zeros of $f(z) = z^3 + 2z = 3$ (5 marks)

(b) The complex number z has modulus 4 and argument $\frac{\pi}{4}$. Find the modulus and argument of

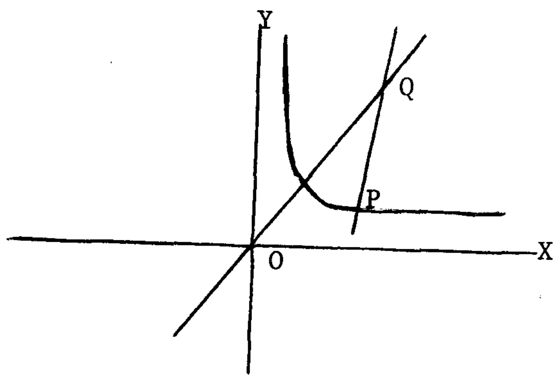
(i) z^2

(ii) iz. (6 marks)

11. (a) Show that the equation of the normal at the point $P(p, \frac{1}{p})$ on the curve xy = 1 is

$$py - p^3x = 1 - p^4$$

(5 marks)



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$2x+6 \quad +4x \quad -9x$

$8x + 6 = 9x$.

$6 = 9x - 8x$

(b) The diagram above shows the normal specified in (a), intersects the line y = x at Q
- (i) Find in a simplified form interms of p, the coordinates of Q
- (ii) Find the equation of the circle with centre P and radius OP, where O is the origin.
- (iii) Verify that the circle obtained in (iii) passes through Q

(6 marks)

12. Carpentry section of one of the technical schools in Tanzania make tables and chairs. The section has two departments. Assembly and Finishing departments. For a particular order from a customer only 48 hours of work are available in the Assembly and 36 hours in the Finishing departments. To manufacture one table it requires 3 hours in the Assembly and 3 hours in the Finishing department; while a chair requires 4 hours in Assembly and 2 hours in Finishing department. If chair fetches Tshs.400 as profit and a table Tshs. 400/=, determine the best combination of tables and chairs to produce so as to get maximum profit.

(11 marks)

13. (a) Solve for x $(4^{x+3})(16^x) = 8^{3x}$

(4 marks)

(b) A linear transformation T sends any point p(x, y) into (x^1, y^1) such that

$x^1 = 2x + 2y$

$y^1 = 2x + 2y$

- (i) Find the matrix which represents the transformation T.
- (ii) Find the image of the point (3, - 2) under linear transformation T.

(4 marks)

(c) Given that $B = \begin{bmatrix} 3 & y-1 \\ y+1 & 1 \end{bmatrix}$ is a singular matrix, find possible values of y. (3 marks)

v 14. (a) Show that $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$. (5 marks)

(b) Let p denote "It is cold" and let q denote "It rains". Write the following statements in symbolic form.
- (i) It rains only if it is cold.
- (ii) It is not cold or it rains.
- (iii) It never rains when it is cold.

(3 marks)

(c) Sketch an electrical circuit to represent the following statement.
$(p \wedge q) \vee (r \wedge s)$

(3 marks)

$3x + 2y = 36$

$\frac{2y}{2} \quad \frac{36-3x}{2}$

$y = \frac{\quad}{2}$

$3x + 4\left(\frac{36-3x}{2}\right) = 48$

$3x + 2(36-3x) = 48$

$\frac{-24}{-3}$

$x = 8$

$y = \frac{36-24}{2}$

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